Evaluate $f'(x)$ using the definition. Let $f(x) = \dfrac{x}{x+1}$

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h} = \frac{\dfrac{x+h}{x+h+1}-\dfrac{x}{x+1}}{h}\right)$$

As it stands, plugging in $h = 0$ evaluates to $\dfrac{0}{0}$, which is indeterminate. Fortunately, we can simplify using basic algebra:

Divide numerators by h: $\dfrac{\frac{x}{h}+1}{x+h+1}-\dfrac{\frac{x}{h}}{x+1}$

Common denominator: $\dfrac{\left(\frac{x}{h}+1\right)(x+1)-\left(\frac{x}{h}\right)(x+h+1)}{(x+h+1)(x+1)}$

Expand numerator: $\dfrac{\left(\frac{x^2}{h}+x+\frac{x}{h}+1\right)-\left(\frac{x^2}{h}+x+\frac{x}{h}\right)}{(x+h+1)(x+1)}$

Simplify numerator: $\dfrac{1}{(x+h+1)(x+1)}$

We can now plug in $h = 0$

$$f'(x) = \lim_{h \to 0}\left(\frac{1}{(x+h+1)(x+1)}\right) = \frac{1}{(x+0+1)(x+1)} = \boldsymbol{\frac{1}{(x+1)^2}}$$